3 Production and efficiency with global technologies*

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WITH APPENDICES BY
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1 Introduction

This chapter considers the problems of production and technology when natural resources or, more generally, non-produced means of production have a relevant role.

The theory of resources, technologies and production, on the one hand, and that of rent, distribution and prices, on the other, both founded on multisectoral structural schemes, have been a constant line of research for me. Firstly, the role of the relative scarcities due to land, natural resources, primary commodities, crops and raw material is analysed (Quadrio Curzio, 1967). This scheme is subsequently enlarged in various directions, examining also a wider category of non-produced means of production, in uniperiodical, comparative uniperiodical and dynamic contexts (Quadrio Curzio, 1975, 1977, 1986; Quadrio Curzio, Manara and Faliva, 1987; Quadrio Curzio and Pellizzari, 1991). In these analyses the original approaches have been improved through the use of more powerful mathematical tools applied also to the new problems. Some of the most important results have been already published in English (Quadrio Curzio, 1980, 1986; Quadrio Curzio and Pellizzari, 1991). Finally, a more general model on resources, technology and rent has recently been published (Quadrio Curzio and Pellizzari, 1996).

The nature and positioning of my approach within general economic theory has been explained in a survey of mine on 'land rent' (Quadrio Curzio, 1987). This category is for me the basis of any kind of rent deriving from 'technological scarcities' due to natural resources or to other 'scarcities'. In that essay the three more important theories of rent are examined (the surplus approach, the marginal productivity approach, the intersectoral approach), pointing out how my theory is a generalisation of the intersectoral rent approach.

The intersectoral rent approach can be linked back, in the history of economic thought, economic history and economic theory, to well-known and fundamental contributions, from Ricardo (1817–21) to Sraffa (1960).

Ricardo's contribution was rather strictly connected to the historical circumstances of the beginning of the last century. Sraffa's contribution, while wide and modern for the structural theory of prices, distribution and technology, was rather limited relative to the analysis of scarce natural resources. This is not only because Sraffa considered mainly 'land', 'corn' and 'crops' but especially because the interconnection between changes in distribution and changes in production were practically neglected both in uniperiodical and in dynamic situations. However, little help in these directions comes from many modern and fundamental dynamic multisectoral theories (see for instance von Neumann, 1945–6; Leontief, 1941, 1951; Pasinetti, 1981) which totally neglect the role of natural resources and technically similar non-produced means of production.

For these reasons I believe that my theoretical contribution, uniperiodical and dynamic, has a high degree of autonomy in the analysis of natural resources, of the non-produced means of production and of technologies in their interconnection with rent, distribution and prices.

As mentioned above, in the present chapter our attention is devoted to the problems of production, changes in activity levels, changes in techniques and in the efficiency of techniques. In the past I approached these problems by means of two theoretical schemes.

The first scheme is based on 'joint' techniques, which give rise to a 'global' technology of the economic system. Particularly important in this case is the use of 'splitting coefficients' and procedures of aggregation and disaggregation of all the processes which employ non-produced means of production. In the present chapter we will take up and widen this scheme.

The second scheme is based on 'disjoint' techniques, with each technique making use of one single non-produced means of production. More than one technique gives rise to 'composite' technologies of the economic system.

There are several reasons for choosing here and widening the global technology approach.

First, it is possible to extend the validity of representations of global multisectoral technology based on 'splitting coefficients' (as well as on technological coefficients) which allow us to examine changes in efficiency when the number of the activated production processes remain unchanged or when the number changes. More generally, three cases can be described: the 'Ricardian' case (diminishing efficiency); the neutral case (constant efficiency); the progressive case (increasing efficiency).

Second, it is possible to highlight further how the nature of 'technological scarcities' is similar, even if not completely identical, to that of 'natural resources scarcities'. In other words, we can maintain that the model takes us well beyond the traditional example of 'land' and 'corn', providing a representation of technologies which adequately fits the situation of contemporary economic systems.

Last but not least, it is possible to formulate a model, which is analytically very complex, in a more compact and synthetic but at the same time more general form. In fact the mathematical treatment with regard to the existence of solutions elaborates a set of instruments that is unusual in multisectoral schemes, which are often limited to the first version of Perron–Frobenius theorems.

This contribution is therefore divided into two parts: in the first part, Quadrio Curzio elaborates the model and the economic line of thought; in the appendices (second part), Manara (appendix 1) and Faliva (appendix 2) carry out the mathematical demonstrations.

2 Data, definitions and hypotheses

In presenting the data, definitions and hypotheses which lie behind this model, we assume that the reader is acquainted with the terminology of linear algebra.[1]

1 Basic commodities In the economic system $m+1$ commodities are produced, which are also the means of production. These basic commodities are of two kinds.

Commodity 1 (RMC1, raw material or commodity 1, an abbreviation we will often use) is a raw material and the quantity in which it is produced is denoted by $q_1(\ldots)$. In its production, non-produced means of production are also utilized in a number, quantity and quality to be determined.

The other m commodities whose quantities are denoted by q_j, for $j=2, \ldots, m+1$, are produced without any direct utilisation of the non-produced means of production.

2 Non-produced means of production (NPMP) and production processes which utilize them. The RMC1 is also produced by NPMP of different qualities up to a maximum number of k. The processes which produce RMC1 can therefore vary in number from 1 to k, and are represented by the vector of technical coefficients

$$[a_1'(h), l_1(h), t_1(h)],\ h=1, 2, \ldots, k \tag{1}$$

with

$$a_1'(h)=[a_{11}(h), \ldots, a_{m+1,1}(h)],\ h=1, \ldots, k$$

The vector of technical coefficients $a_1(h)$ represents the inputs of $m+1$ commodities for producing the RMC1 through the utilisation of NPMP of type h. These coefficients also include quantities of 'necessary consumption' for each commodity (see the following point 4); $l_1(h)$ are the technical labour coefficients; $t_1(h)$ are the technical coefficients of NPMP of type h. Each vector $a_1(h)$ is such as to permit the viability, to the extent to which it concerns, of the technique of the successive type (4). With $T_1(h)$ denoting the available quantities of NPMP of type h, the quantity of RMC1 of process h is subject to the constraint

$$q_1(h)t_1(h) \leq T_1(h) \tag{2a}$$

From (2a), it follows that the maximum quantity of $q_1(h)$ which can be produced is equal to

$$\bar{q}_1(h) = T_1(h)/t_1(h) \tag{2b}$$

3 Processes of production for commodities j, that is, those which do not directly utilise NPMP. Each of these is produced by a single process

$$[a_j'; l_j]$$

with

$$a_j' = [a_{1j}, \ldots, a_{m+1j}], j=2, \ldots, m+1 \tag{3}$$

the vector of the usual input–output coefficients, augmented by quotas of necessary consumption. These processes give rise to viable techniques (of the following type (4)).

4 Technical coefficients and levels of necessary consumption Each coefficient of the vectors $a_1(h)$ and a_j is augmented with respect to those of Leontief by levels of necessary consumption, that is, by amounts of each single commodity included in a conventional bundle of necessary consumption. In this way, every reference to final consumption demand can be avoided, if necessary, when a process of growth is analysed.

5 Production techniques Since the $m+1$ commodities are of a basic kind, all the processes (3) and at least one process (1) must be utilised to produce them. Since there are k viable processes of type (1), k distinct production techniques can be identified which differ only in terms of the process which produces commodity 1. Formally, each technique is represented by the matrix

$$A(h) = [a_1(h); a_2, \ldots, a_{m+1}] \geq O \tag{4}$$

together with the vector

$$L'(h) = [l_1(h); l_2, \ldots, l_{m+1}] \geq O$$

and with the constraints

$$t_1(h) \leq T_1(h)/q_1(h)$$

with $h=1, \ldots, k$.

6 Viability of the techniques Each technique (matrix) $A(h)$ is non-negative, indecomposable and viable; it admits therefore a uniform rate of positive net product, or a maximum rate of positive profit (or an eigenvalue which is maximum, real, positive, not repeated and less than 1).

7 Scale constraints of the techniques Because of the limited quantities available of each NPMP of type h, and therefore because of constraint (2a) on $q_1(h)$, each technique (4) cannot produce more than a certain quantity of all the $m+1$ commodities since these are all basic.

8 Production technology The constraints of scale imposed on each technique by the limited quantities available of NPMP of type h may demand the activation of more techniques $A(h)$ in order to satisfy certain production goals. The simultaneous operation of several $A(h)$ techniques can be analysed by two distinct analytical methods: that of 'global technologies' in which the techniques $A(h)$ are 'joined' in a particular manner, and that of 'composite technologies' in which the $A(h)$ techniques remain 'disjointed', but nevertheless connected. As mentioned above, here we will utilise global technologies.

9 Orders of efficiency between techniques and non-produced means of production The differences between techniques (4) are caused solely by the process which produces RMC1. To establish an order of efficiency between the techniques (4) means, therefore, establishing an order of efficiency between the k non-produced means of production which cannot be compared in terms of 'corn per hectare', given the complexity of the processes in which they are contained.

For each technique (4), a price-distribution 'economic subsystem' (thus defined because the name 'economic system' is reserved for those technologies which contain several processes with NPMP) can be constructed as follows

$$[1+\pi(h)]A(h)p(h)+L(h)w(h)=p(h),\ h=1, \ldots, k \tag{5}$$

where $\pi(h)$ is the rate of profit, $w(h)$ the unitary wage (both uniform for each process) and $p(h)$ the price vector of the $m+1$ commodities.

For w, fixed exogenously, the order of efficiency of the k subsystem and therefore of the k non-produced means of production is given by the succession

$$\pi(1)>\pi(2)>\ldots>\pi(k) \tag{6a}$$

with the h indexes suitably permuted.

If $w=0$, the order of efficiency (6a) coincides with that given by the succession of the uniform rate of net product of each technique $A(h)$.

$$s(1)>s(2)>\ldots>s(k) \tag{6b}$$

If $w>0$, the succession (6a) and (6b) might not coincide, and, in fact, might even be reversed. In this case, the indices h of (6b) have been opportunely permuted with respect to those of (6a). The order (6a) can be called the 'price-distribution or uniperiodal' order since, by following it, there is no guarantee that the techniques $A(h)$ will be activated according to a succession of maximum growth potential. This is assured, instead, for each technique by following the order (6b) which can consequently be called the 'order of physical or dynamic efficiency'.

10 Succession of the eigencoefficients Let us assume, without losing anything in generality, that

$$a_{11}(1)\leq a_{11}(2)\leq\ldots<a_{11}(k) \tag{7}$$

since this is one condition sufficient for the existence of solutions when moving from aggregated to disaggregated matrices (see the following section 8, and appendix 1).

This assumption is not particularly restrictive. If the physical efficiency order is followed, it is a reasonably obvious assumption, even if it is not necessary. This does not imply, on the other hand, that the price-distribution order of efficiency coincides with the physical order.

11 Dimensions of the economic system These can be identified in various ways and on the basis of various criteria. One of these is the dimension of the technology, or the number of processes operating with NPMP. In this sense, it can be said that an economic system with one NPMP is 'smaller' than a system operating with two or more kinds of NPMP.

3 The problems

Our aim is to analyse:

1 The activity levels of the economic system and changes in these levels. This means determining the number of NPMP which are operating, the production processes and employment.
2 The technology of the economic system. When more than one process with NPMP is started up, the technology becomes a complex entity since it consists of more than one technique of type (4).

3 The changes in the technologies which can vary from changes in the activity levels, from the choice of techniques and from technical progress.
4 The changes in the uniperiodical physical efficiency of the economic system.

4 The economic system with one technique

Let us consider the smallest economic system in which there is only one technique operating, in other words, that system which includes the most efficient technique, on the basis of any one of the two orders of efficiency described above, among the k processes with NPMP. The production system which corresponds to a uniform rate of net product is given by

$$[(1+s(1))A(1)-I]q(1)=O \tag{8a}$$

$$[A(1)-\tilde{\lambda}(1)I]q(1)=O \tag{8b}$$

with

$$0<\tilde{\lambda}(1)=1/(1+s(1))<1 \tag{9}$$

$$q_1(1)\leq\bar{q}_1(1)=T_1(1)/t_1(1) \tag{10}$$

$$L'(1)q(1)=L\leq\bar{L} \tag{11}$$

With $s(1)$ we denote the uniform rate of net product; with $\tilde{\lambda}(1)$ the maximum eigenvalue of $A(1)$; with $q(1)$ the eigenvector of the production processes related to $\tilde{\lambda}(1)$; with $\bar{L}$ the existing labour force. Since $A(1)$ is non-negative, indecomposable and viable, the eigenvalue $\tilde{\lambda}(1)$ is real, positive, not repeated and less than one; the eigenvector $q(1)$ is strictly positive and structurally defined while its scale is determined on the basis of (10) (as an equality) or of (11).

Given that h can assume the values 1, ..., k, by substituting in $A(1)$ the vector $a_1(h)$ for the vector $a_1(1)$, other $k-1$ subsystems or techniques (8), (9) and (10) can be constructed. Each of these is autonomous, apart from (11), with a maximum limit of production given by (10). Each of these techniques taken individually can grow at a maximum rate $s(h)$ within the limits given by (10) or (11). The problems of changes in the activity levels and of dynamics become, however, more complex when the techniques must be 'chained' due to the constraints of the NPMP.

Let us go back to the relations (8), (9) and (11) and consider (11) as an equality. If the resulting vector of the production processes, that is, the vector which gives full employment of labour violates (10), then NPMP of type 1 is insufficient to allow the full utilisation of labour. If we suppose that this is the case, then it will be necessary also to start up the process 2 which utilises NPMP of type 2.

5 Global technologies and splitting coefficients

The economic system must therefore utilise two processes with NPMP, as well as all the other m processes. The global technology which represents this situation is given by the matrix

$$A_\alpha(1,2)=\begin{bmatrix} a_{11}(1) & 0 & \alpha_{12}(1) & \vdots & \alpha_{1,m+1}(1) \\ 0 & a_{11}(2) & \alpha_{12}(2) & \vdots & \alpha_{1,m+1}(2) \\ a_{21}(1) & a_{21}(2) & a_{22} & \vdots & a_{2,m+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m+1,1}(1) & a_{m+1,1}(2) & a_{m.+1,2} & \vdots & a_{m+1,m+1} \end{bmatrix} \tag{12}$$

The two processes $a_1(1)$ and $a_1(2)$ are now *jointed* in the production of RMC1 which is required by the whole economic system. The 'splitting or splitted supply coefficients' (α) identify the extent to which processes 1 and 2, respectively, supply RMC1 to every other process of the economic system. Elaborating them we therefore have

$$\alpha_{1j}(1)+\alpha_{1j}(2)=a_{1j}$$

The increase of $\alpha_{1j}(2)$ with respect to $\alpha_{1j}(1)$, which correspondingly falls, therefore indicates that process $a_1(2)$ assumes a greater relative weight in the economic system as it increasingly replaces process $a_1(1)$ in the supply of RMC1 compared with all the other processes in the economic system. In turn, the relative weight of this process in the economic system is reduced, even if it remains activated at its maximum level, which corresponds to the full utilisation of $T_1(1)$ (see (10) above).

It should be noted that in $A_\alpha(1, 2)$ those coefficients which could be defined as 'reciprocal splitting coefficients', that is, those which refer to the outputs supplied by process 1 to process 2, and vice versa, have not been inserted. The reason is evident: if the activity level of the economic system was so high that all the $\alpha_{1j}(1)$ were reduced to almost zero, process 1 would still produce a quantity of $q_1(1)$ sufficient for its own utilisation, given that $a_{11}(1)<1$. Supplying process 1 with RMC1 coming from process 2, that is, introducing $\alpha_{11}(2)>0$, means that an amount of RMC1 produced and already utilised by process 1 is released and allocated for utilisation in one or more of the j processes. Compensation (and not a substitution) would therefore occur which would prevent the operation from having any influence on the activity levels. The same reasoning can be carried out for the reciprocal splitting coefficients of process 1 with respect to 2.

The global technology $A_\alpha(1, 2)$ changes when there are changes in the α coefficients, is different, in part, from the leontievian matrices and represents, in our opinion, an innovation in the representation of production systems by including in the same matrix more than one process which produces the same commodity and other processes which produce different commodities. The matrices of type $A_\alpha(1, 2)$ have the following properties:

(a) they are non-negative;
(b) they are indecomposable (except in extreme theoretical cases when they assume the values of the splitting coefficients);
(c) they are viable, that is, they admit a uniform rate of net positive product (9). This property derives from relation (9);
(d) they are non-singular. We exclude processes which are linearly dependent.

6 The economic system with two techniques

The unknown values of the economic system, that is, α, s, q and L are determined by resolving the system

$$[(1+s_\alpha(1, 2))A_\alpha(1, 2)-I]q_\alpha(1, 2)=O \tag{13}$$

with respect to the conditions

$$\alpha_{1j}(1)+\alpha_{1j}(2)=a_{1j};\ \alpha_{1j}(1)>0,\ \alpha_{1j}(2)>0;\ j=2, \ldots, m+1 \tag{14}$$

$$\bar{q}_1(1)=T_1(1)/t_1(1) \tag{15}$$

$$q_1(1)=\bar{q}_1(1) \tag{16}$$

$$q_1(2)\leq\bar{q}_1(2)=T_1(2)/t_1(2) \tag{17}$$

$$L'(1, 2)q_\alpha(1, 2)=L\leq\bar{L} \tag{18}$$

given that

$$L'(1, 2)=[l_1(1), l_1(2), l_2, \ldots, l_{m+1}]$$
$$q'_\alpha(1, 2)=[q_1(1), q_1(2), q_2, \ldots, q_{m+1}] \tag{19}$$

The solution can be found be increasing $\alpha_{1j}(2)$ in such a way that the constraint given by the full utilisation of NPMP of type 1, that is, by (16), does not prevent the production processes $q_\alpha(1, 2)$ from growing to that level at which (17) is satisfied as an equality, that is, up to the full utilisation of NPMP of type 2. At that level of $q_\alpha(1, 2)$, three alternatives can be given for the labour force:

(a) it is fully employed and (18) is therefore satisfied as an equality;

(b) it is insufficient for the full utilisation of NPMP of type 2, and (18) is, therefore, violated. In this case, by reducing the $\alpha_{1j}(2)$, $q_\alpha(1, 2)$ will be reduced so that (18) will become an equality;

(c) it is partially unutilised and (18) is therefore satisfied as an inequality. In this case, the economic system must expand by introducing a further NPMP, that of type 3.

We have required that the economy based on a joint technology has production processes which are structured in such a way as to generate a uniform rate of net production.

The meaning of $s_\alpha(1, 2)$ in this situation does not altogether coincide, however, with that of $s(h)$. Both $s_\alpha(1, 2)$ and $s(h)$ are indicators of the uniperiodal efficiency of the respective economic systems; $s(h)$ is also the maximum and constant growth rate of technique h; $s_\alpha(1, 2)$ is connected only to the growth rate of technology $A_\alpha(1, 2)$ and, moreover, is not constant.

The comparison between the two cases will become interesting for clarifying the differences between the dynamics with and without NPMP.

7 The economic system with k techniques

The number of processes operating with NPMP will depend, on the basis of what has been said, on the quantity constraints of the NPMP themselves and labour. The widest available global technology is that in which k processes with NPMP operate. This technology is given by the following matrix of order $(k+m)$ which has all the properties defined in section 2 above, and in which the processes from 1 to k produce RMC1, and the processes from 2 to $m+1$ produce the other m commodities. Such a technology will be employed, following the previously chosen order of efficiency, when the first $k-1$ processes with NPMP are utilised at their maximum level established by the quantity available of NPMP.

$$A_\alpha(1, \ldots, k) = \left[\begin{array}{ccccc|ccc}
a_{11}(1) & 0 & 0 \ldots & 0 & 0 & \alpha_{12}(1) & \alpha_{13}(1) & \ldots\alpha_{1,m+1}(1) \\
0 & a_{11}(2) & 0 \ldots & 0 & 0 & \alpha_{12}(2) & \alpha_{13}(2) & \ldots\alpha_{1,m+1}(2) \\
\ldots & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 \ldots & a_{11}(k-1) & 0 & \alpha_{12}(k-1) & \alpha_{13}(k-1) & \ldots\alpha_{1,m+1}(k-1) \\
0 & 0 & 0 \ldots & 0 & a_{11}(k) & \alpha_{12}(k) & \alpha_{13}(k) & \ldots\alpha_{1,m+1}(k) \\
\hline
a_{21}(1) & a_{21}(2) & \ldots \ldots & a_{21}(k-1) & a_{21}(k) & a_{22} & a_{23} & \ldots a_{2,m+1} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
a_{m+1,1}(1) & a_{m+1,1}(2) & \ldots \ldots & a_{m+1,1}(k-1) & a_{m+1,1}(k) & a_{m+1,2} & a_{m+1,3} & \ldots a_{m+1,m+1}
\end{array}\right] \qquad (20)$$

The economic system as a whole is described by the following relations

$$[(1+s_\alpha(1,\ldots,k))A_\alpha(1,\ldots,k)-I]q_\alpha(1,\ldots,k)=O \quad (21)$$

$$\sum_{h=1}^{k}\alpha_{1j}(h)=\alpha_{1j};$$

$$\alpha_{1j}(h)>0, j=2,\ldots,m+1; h=1,\ldots,k \quad (22)$$

$$\bar{q}_1(h)=T_1(h)/t_1(h) \quad (23)$$

$$q_1(h)=\bar{q}_1(h), h=1,\ldots,k-1 \quad (24)$$

$$q_1(k)\leq\bar{q}_1(k) \quad (25)$$

$$L'(1,\ldots,k)q_\alpha(1,\ldots,k)=\bar{L} \quad (26)$$

with

$$L'(1,\ldots,k)=[l_1(1),\ldots,l_1(k),l_2,\ldots,l_{m+1}]$$

$$q'_\alpha(1,\ldots,k)=[q_1(1),\ldots,q_1(k),q_2,\ldots,q_{m+1}]$$

Alternatively, instead of (25) and (26), the following relations can be considered

$$q_1(k)=\bar{q}_1(k) \quad (27)$$

$$L'(1,\ldots,k)q_\alpha(1,\ldots,k)=\bar{L} \quad (28)$$

There are three kinds of possible solution to this system.

(a) The solution of system (21)–(26) identifies the uniform rate of net product and the total production processes which correspond to the full utilisation of labour. The NPMP are completely utilised apart from the kth which cannot be completely utilised due to the labour constraint.
(b) The solution of system (21)–(24), (27) and (28) identifies instead the case in which NPMP are completely utilised, whereas, due to the constraints imposed by the availability of NPMP, labour is not completely utilised.
(c) The solution of system (21)–(24), (26) and (27) identifies the case of full employment of both the NPMP and labour.

This approach makes it possible, therefore, to establish how technology changes when there are two categories of limited means of production: 'labour' and 'land'. In turn, in the absence of technical progress, changes in technology can be of two kinds.

The first are those linked to an increase in the weight of the last process with NPMP activated, necessitated by an increase in the available labour force. In this case, the net product rate and outputs change, but not the processes.

The other changes are those which lead to an increase in the number of processes with NPMP activated. In this situation, changes in the dimensions of the global technology are also added to the changes occurring in the first case.

8 The aggregated and disaggregated economic system

The solution of system (21)–(26), or of system (21)–(24), (27) and (28), or of system (21)–(24), (26) and (27), as formulated, is not always easy with the usual procedures for calculating eigenvectors and eigenvalues. Constraints on the vector are at least $k-1$, as can be seen from (23) and (24), whereas the eigenvector of quantities can be defined, in most normal cases, by means of only one constraint.

However, having taken into account the economic significance of the problem, the first $k-1$ processes – activated at the maximum level – can be aggregated into one unique process. This approach consists precisely in introducing first the splitting coefficients as well as the disaggregated global technologies, and then passing to aggregated technologies (and vice versa), as can be immediately seen by defining

$$(a_1^*(k-1))' = [a_{21}^*(k-1), \ldots, a_{m+1,1}^*(k-1)] \tag{29}$$

$$a_{i1}^*(k-1) = \sum_{\nu=1}^{k-1} a_{i1}(\nu)\beta(\nu), \; i=1, \ldots, m+1$$

$$l_1^*(k-1) = \sum_{\nu=1}^{k-1} l_1(\nu)\beta(\nu) \tag{30}$$

$$\beta(\nu) = \bar{q}_1(\nu)/\bar{q}_1^*(k-1), \; \sum_{\nu=1}^{k-1}\beta(\nu) = 1 \tag{31}$$

$$\bar{q}_1^*(k-1) = \sum_{\nu=1}^{k-1} \bar{q}_1(\nu) \tag{32}$$

The problem of the production scale of the economy depends, therefore, on the relative weight of the $k-1$ aggregated processes with respect to the weight of process k, with all processes utilising NPMP.

The determination of the splitting coefficient can be related, therefore, on the one hand, to the process obtained through an aggregation of the first $k-1$ processes, and , on the other hand, to the kth process. The splitting coefficients shown in the same column can be summed in that they are multiplied by the same quantity and concern the same commodity. At this point, it is opportune, especially for the elaborations which follow, to introduce the following notations.

$$(\alpha^*(k-1))'=[\alpha^*_{12}(k-1), \ldots, \alpha^*_{1,m+1}(k-1)]$$
$$\alpha^*_{1j}(k-1)=\sum_{\nu=1}^{k-1}\alpha_{1j}(\nu), j=2, \ldots, m+1$$
$$\alpha'=[\alpha_1, \alpha_2, \ldots, \alpha_m]$$
$$\alpha_i=\alpha_{1,i+1}(k), i=1, 2, \ldots, m$$
$$a^*_1=\alpha^*(k-1)+\alpha \tag{33}$$

By means of these aggregations, the global technology can once again be described by a matrix of order $m+2$, as in the case of the economic system with only two techniques. The aggregated global technology is therefore given by

$$A^*_\alpha(k-1, k)=\left[\begin{array}{c|c|ccc} a^*_{11}(k-1) & 0 & \alpha^*_{12}(k-1) & \alpha^*_{13}(k-1) & \ldots\alpha^*_{1,m+1}(k-1) \\ \hline 0 & a_{11}(k) & \alpha_{12}(k) & \alpha_{13}(k) & \ldots\alpha_{2,m+1}(k) \\ \hline a^*_{21}(k-1) & a_{21}(k) & a_{22} & a_{23} & \ldots a_{2,m+1} \\ a^*_{31}(k-1) & a_{31}(k) & a_{32} & a_{33} & \ldots a_{3,m+1} \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots \\ a^*_{m+1,1}(k-1) & a_{m+1,1}(k) & a_{m+1,2} & a_{m+1,3} & \ldots a_{m+1,m+1} \end{array}\right] \tag{34a}$$

In order to analyse the changes in efficiency of the activity levels of the economic system in the following sections, it is opportune to rewrite immediately the above matrix also in a compact form

$$A^*_\alpha(k-1, k)=\begin{bmatrix} a^*_{11}(k-1) & 0 & (a^*_1-\alpha)' \\ 0 & a_{11}(k-1) & \alpha' \\ a^*_1(k-1) & a_1(k) & A(m) \end{bmatrix} \tag{34b}$$

Having denoted the submatrix formed by the last m rows and columns of (34a) with $A(m)$, and denoted the vector formed by the elements of (34a) in the last m rows of the second column with $a_1(k)$.

The aggregated economic system is therefore given by

$$\begin{aligned} &[(1+s_\alpha)A^*_\alpha(k-1, k)-I]q^*_\alpha(k-1, k)=O \\ &[A^*_\alpha(k-1, k)-\tilde{\lambda}_\alpha I]q^*_\alpha(k-1, k)=O \end{aligned} \tag{35}$$

with

$$s_\alpha=(1/\tilde{\lambda}_a)-1; 0<\tilde{\lambda}_a<1$$

$$(L^*(k-1, k))' = [l_1^*(k-1), l_1(k), l_2, \ldots, l_{m+1}] \quad (36)$$

subordinated to the constraints (22), (25), (27) and (32) and to the constraint

$$(L^*(k-1, k)), q_\alpha^*(k-1, k) \leq \bar{L} \quad (37)$$

In the formulation there are also three alternative kinds of solution:

(a) The first solution allows us to identify a vector $\alpha^*(k-1)$, a vector $\alpha(k)$, a quantities vector $q_\alpha^*(k-1, k)$, and a rate of net product s_α, which give the full employment of labour, whereas NPMP of type k remain partially unemployed.
(b) The second solution will give NPMP of type k fully employed with labour unemployed.
(c) Finally, the third solution will give full utilisation of NPMP and labour.

The procedure for solving this system, in economic terms, is the following: if the α, which are related to $q_\alpha^*(k-1, k)$ and to s_α which satisfy the constraints of full utilisation of the first $k-1$ NPMP, do not give full utilisation of labour, $\alpha^*(k-1)$ must be reduced with a corresponding increase in $\alpha(k)$. In this way, the constraint of the first $k-1$ NPMP is weakened by increasing the weight of the process k and this thereby increases all the quantities produced which incorporate the NPMP as a mean of production. This situation of $\alpha^*(k-1)$ and the corresponding increase in $\alpha(k)$ is arrested as soon as the constraint (27) or (37) becomes operative. The problem of disaggregation consists in splitting the vector $\alpha^*(k-1)$ between the $k-1$ processes in order to pass the matrix (34), $A_\alpha^*(k-1, k)$, to the matrix (20), $A_\alpha(1, \ldots, k)$, without modifying the solutions obtained.

The mathematical solution is given in appendix 1. This solution determines the splitting coefficients with the distribution of $\alpha_{1j}^*(k-1)$ among the $k-1$ processes with NPMP as follows

$$\alpha_{1j}(\nu) = \alpha_{1j}^*(k-1)\beta(\nu);\ \nu = 1, \ldots, k-1;\ j = 2, \ldots, m+1 \quad (38)$$

Matrix (20), $A_\alpha(1, \ldots, k)$, in the north-east submatrix will therefore have $k-1$ row vectors

$$[\alpha_{12}^*(k-1)\beta(\nu), \alpha_{13}^*(k-1)\beta(\nu), \ldots, \alpha_{1,m+1}^*(k-1)\beta(\nu)] \quad (39)$$

It should be noted that a distribution of $\alpha_{1j}^*(k-1)$ with non-uniform weights $\beta(\nu)$ in every row is completely useless in that it would imply compensations imposed by (21) and (22). By simplifying for a case with three commodities and two processes with NPMP, it can be shown that if the process with NPMP 1 increases the share of 'corn' supplied to the 'iron' sector, that is, if $\beta_{12}(1)$ is increased above $\beta(1)$, the weight of 'corn' supplied to the 'services' sector must diminish correspondingly, that is, $\beta_{13}(1)$ falls below $\beta(1)$. At the same time, the share of 'corn' supplied to the 'iron' sector by the process with NPMP of type 2 will have to diminish, that is, $\beta_{12}(2)$ will fall below $\beta(2)$, and, correspondingly, the share of 'corn' supplied to the 'services' sector will have to increase, that is, $\beta_{13}(2)$ will rise above $\beta(2)$.

9 Changes in efficiency and activity level

Let us now suppose that the economic system is in a situation of full labour employment, with a number of k NPMP activated, but with the kth not completely utilised. If the labour force increases, it becomes necessary, if an increase in employment is desired, to increase further the utilisation of NPMP of type k. This gives rise to the following results:

(a) a structural change in the global technology whose dimensions, however, do not change, and continue to consist of k processes with NPMP (and of the other m processes);
(b) a continuous change in the efficiency of the global technology in that the k process assumes a greater weight within the technology;
(c) an increase in the quantities produced and in employment.

This is the case of continuous changes in efficiency.

If the increase in the labour force requires also the utilisation of NPMP successive to the kth, which we now suppose exists and that, to simplify, we denote with $k+1$, we will have:

(d) a change in the structure and dimensions of the global technology;
(e) a change, with a discontinuity, in the efficiency of the global technology;
(f) an increase in the quantities produced and in employment.

This is the case of the discontinuous changes in efficiency.

Let us examine the two cases without, however, carrying out a dynamic analysis, that is, without considering the accumulation of the net product. We will restrict our attention for now to comparisons between uniperiodal situations.

10 Changes in efficiency

By taking the uniform rate of net production among the different indicators of the efficiency of the economic system, we can examine its changes as the level of activity increases, by extending, that is, the utilisation of NPMP of type k, employment and production processes.

The mathematical demonstrations for this and the following section can be found in appendix 2. In relation to system (35), and therefore to (34) and to (36), and omitting, for simplicity, the magnitudes $k-1$ and k, we have

$$\frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} = \frac{p^*_\alpha \frac{\partial A^*_\alpha}{\partial \alpha_{1j}(k)} q^*_\alpha}{(p^*_\alpha)' q^*_\alpha} \tag{40}$$

where with $(p^*_\alpha)' = [p^*_1(k-1),\ p_1(k),\ p_2,\ \ldots,\ p_{m+1}]$ we denote the left-hand eigenvector of the matrix A^*_α.

This derivation expresses the effect on the net production rate (that is, on the maximum eigenvalue) of A^*_α, of an increase in the splitting coefficients relating to the process with NPMP of type k and therefore the increase in the weight of this process in the economic system.

The mathematical elaboration of (40) gives rise to the following result

$$\frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} = \frac{1}{(p^*_\alpha)' q^*_\alpha}[p_1(k) - p^*_1(k-1)] q_j;\ j=2, \ldots, m+1 \tag{41}$$

From (40) and (41) we can derive three alternative cases:

(a) We have the case of increasing efficiency when

$$p_1(k) < p^*_1(k-1) \tag{42}$$

which gives

$$\frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} < 0 \text{ or } \frac{\partial s_\alpha}{\partial \alpha_{1j}(k)} > 0 \tag{43}$$

(b) We have the case of constant efficiency when

$$p_1(k) = p^*_1(k-1) \tag{44}$$

which gives

$$\frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} = 0 \text{ or } \frac{\partial s_\alpha}{\partial \alpha_{1j}(k)} = 0 \tag{45}$$

(c) We have the case of decreasing efficiency when

$$p_1(k) > p^*_1(k-1) \tag{46}$$

which gives

$$\frac{\partial \tilde{\lambda}_{\alpha}}{\partial \alpha_{1j}(k)} > 0 \text{ or } \frac{\partial s_{\alpha}}{\partial \alpha_{1j}(k)} < 0 \qquad (47)$$

The identification of the three cases depends, therefore, on the relations of magnitude between $p_1^*(k-1)$ and $p_1(k)$ which represent the prices of RMC1 (of 'corn') produced respectively by the aggregated process $k-1$ and by the process k.

If the vector p_α^* is the left-hand eigenvector of the matrix A_α^*, it is the eigenvector of the prices related to an assumed maximum rate of profit for the aggregated global technology. That is

$$(p_\alpha^*)' A_\alpha^* = \tilde{\lambda}_\alpha (p_\alpha^*)' \qquad (48)$$

The prices considered here and the assumed profit rate are not, however, those of the price-distribution system related to the global technology A_α. In this case, the price of RMC1 is unique for the whole economy and is given by the least efficient technique (cf. points 5 and 9, section 2 above), that is, the technique which incorporates the least efficient process with NPMP. This technique also determines the rate of profit. The other processes included in the technology, which produce RMC1, and which are more efficient, will have, as a consequence, a rent. In the present case, instead, it is as if the two processes which generate RMC1 produce two commodities which are different and which have two different prices. The assumed profit rate is therefore different from that which corresponds to the usual price-distribution system, with the absence of rent, in this approach, which has a purely instrumental role in the quantities system.

These prices are therefore represented by: $p_1^*(k-1)$ the price based on the cost of production, given by the circulating capital, utilised by the aggregated process $k-1$; $p_1(k)$ the price of the process with NPMP of type k. These can be considered indicators of the produced means of production utilised by the two processes, aggregated and disaggregated, with NPMP.

We therefore have increasing efficiency when $p_1^*(k-1) > p_1(k)$, that is, when we increase the weight of the process which utilises less circulating capital per unit of output. Correspondingly, we have constant efficiency in the case of $p_1^*(k-1) = p_1(k)$, and decreasing efficiency in the case of $p_1^*(k-1) < p_1(k)$.

All three cases are possible, depending, alternatively, on the order of efficiency.

If the order of physical efficiency is followed (6b) – the level of w permitting – we will have decreasing efficiency both when the dimensions of the global technology do not change, and when its dimensions increase. In this case, when another process with NPMP is introduced, the uniform rate of net product falls discontinuously. We could claim this to be an updated version of the Ricardian case. The expansion process of the economy will be arrested when all the viable NPMP have been utilised (see section 2, point 6). This means that if we introduced a further NPMP, the technique A, constructed with this last process (see (8)), would be such that

$$A(k+x)q(k+x) \geq q(k+x),\ x \geq 1 \tag{49}$$

and therefore with a zero or negative rate of net product.

The price–distribution order of efficiency can, however, also give rise to the opposite case so that the techniques $A(h)$ are arranged in such a way that their uniform rates of net product are seen to be increasing.

This situation could be the basis, whenever we are interested in the historical phases of economic development which are not considered here, for analysing a technical progress which saves circulating capital, that is, which only affects the 'new' processes with NPMP not yet activated, and which does not determine – at least within a certain period of time – the disactivation of the 'old' processes. In this case, naturally, also the historical order of price–distribution efficiency should give an increasing efficiency of the processes.

This observation allows us the opportunity to observe that a wider analysis of the problems of the order of efficiency, which does not concern, however, our present purpose, ought to allow the identification of historical orders of efficiency, with or without technological progress, and the logical orders of efficiency, which are those which interest us here. The price–distribution order of efficiency, finally, can give rise to alternating trends in the uniform rates of the net product of the global technology as other processes with NPMP are introduced.

This occurs because a non-univocal order of uniform rates of net product can be related to a univocal order (6a).

11 Growth in activity levels

This is the other aspect to be considered, both in the continuous and discontinuous case.

Let us denote with

$$\tilde{\tilde{q}}' = [\tilde{\tilde{q}}_1(k), \tilde{\tilde{q}}_2, \ldots, \tilde{\tilde{q}}_{m+1}] \tag{50}$$

the subvector consisting of the last $m+1$ elements of the vector q^*_α, defined by (36) above, and normalised with the first element set equal to 1. It can then be demonstrated that

$$\frac{\partial \tilde{\tilde{q}}_\alpha}{\partial \alpha_{1j}(k)} > 0 \tag{51}$$

whenever

$$\frac{\partial s_\alpha}{\partial \alpha_{1j}(k)} \geq 0; \ \frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} \leq 0; \ p_1^*(k-1) \geq p_1(k)$$

Therefore, when, with greater activation of the process with NPMP of type k, the efficiency of the economic system grows (or remains constant), the quantities produced also increase.

If the efficiency of the economic system diminishes

$$\frac{\partial s_\alpha}{\partial \alpha_{1j}(k)} < 0; \ \frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_{1j}(k)} > 0; \ p_1^*(k-1) < p_1(k) \tag{52}$$

the mathematical analysis (see appendix 2) shows that the production processes will increase only if $p_1(k)$ does not exceed the amount $p_1^*(k-1)$ beyond certain levels.

The economic interpretation of this condition is not immediately evident on the basis of the mathematical formulation, and probably requires an analysis in terms of vertically integrated sectors. The extreme case should, however, be the following: if the process with NPMP of type k is so inefficient that it generates a negative rate of net product (which is why the subsystem (8b) constructed with this process would have $\tilde{\lambda}_\alpha > 1$), its increased weight in the economic system, with the growth of α, would involve, at a certain point, reducing (or maintaining constant) production processes precisely because the absorption of the means of production exceeds (or equals) the production processes themselves. This situation will never be reached, however, because such a process k, by violating (9), will never be activated.

There is also a second element which supports the conclusion of an increase in production processes also for the case considered by (52). On the basis of (18) of appendix 2, the change in the subvector $\tilde{\tilde{q}}_\alpha$ of the normalised quantities is seen to be linked to the change in the eigenvector $\tilde{\lambda}_\alpha$ by the simple relation

$$\frac{\partial \tilde{\lambda}_\alpha}{\partial \alpha_j} = \frac{\partial \beta'_\alpha}{\partial \alpha_j} \tilde{\tilde{q}}_\alpha + \beta'_\alpha \frac{\partial \tilde{\tilde{q}}_\alpha}{\partial \alpha_j} \tag{53}$$

Taking account of (16) in the same section, it is easy to verify that (53) can be expressed as

$$\frac{\partial\tilde{\lambda}_\alpha}{\partial\alpha_j}=[0, -e'_{j(m)}].\tilde{\tilde{q}}_\alpha+[0, (a_1^*-\alpha)'].\frac{\partial\tilde{\tilde{q}}_\alpha}{\partial\alpha_j} \tag{54}$$

By inspecting (54), it can be deduced, therefore, that if we assume that (52) holds, we must have

$$[0, (a_1^*-\alpha)'].\frac{\partial\tilde{\tilde{q}}_\alpha}{\partial\alpha_j}>0 \tag{55}$$

The increase in a splitting coefficient implies, therefore, that the utilisation of RMC1 as a mean of production, generated by the aggregated process $k-1$, correspondingly increases with the reduction in the net product of that process which produces 'non-augmentable corn'. It is evident that the presupposition, for increasing the utilisation of RMC1 as a means of production, is that the production of at least one of the other m 'industrial' sector processes (as distinct from the two 'agricultural' processes, $k-1$ and k) increases and therefore its requirement of 'corn' will increase. Given that the economic system is indecomposable, it is plausible that the increase in production of the industrial sector and of the agricultural sector k (in which it can be analytically shown that production always increases as α_j grows) induces increases in production in all the other sectors, excluding, obviously, the aggregated 'agricultural' sector $k-1$, which has a fixed output. In all of the sectors, including that of agriculture with variable output, additional means of production, which must be produced, will be required. Finally, compensations with a reduction in those production processes which depend on sector $k-1$ should not occur since this sector's output is fixed.

12 Conclusions

In this chapter we have determined all magnitudes of the quantity system (productions, net products and so on) and their changes by using a 'global technology' which is founded on a scheme of 'joint techniques'.

The main feature of such a technology consists in representing, through a unique matrix, both Leontief-type production processes and production processes based on splitting coefficients regarding the same 'primary commodity' using different NPMP.

On this basis, we have examined two main cases: the case of continuous changes in the efficiency of technology and the case of discontinuous changes in efficiency.

In the first case, the number of processes with activated NPMP does not change; however, the global technology undergoes changes in structure (though not in dimension) and in efficiency as the last activated process with a NPMP takes an increasing weight. These changes take place in the continuum through variations in the 'splitting coefficients', which are determined by splitting input–output coefficients with special analytical tools.

The second case is characterised by discontinuous changes in efficiency because the number of processes with NPMP increases. Then the structural dimensions and efficiency of technology change.

In both cases, our analysis probes into three possibilities: increasing efficiency, constant efficiency and decreasing efficiency. Having chosen the uniform rate of net product of a technology as the index of efficiency, the three above possibilities depend upon the relationship between the price–distribution order of efficiency and the physical order of efficiency among NPMP. The price-distribution order of efficiency is the one among rates of profit (when the unitary wage is given) or among unitary wages (when the rate of profit is given), where each one of these distributive magnitudes is associated with one NPMP. In other words it is the order among the different techniques where each technique includes one NPMP only. The physical order of efficiency is the one among the NPMP (or among the techniques constructed on them) and it is established on the basis of the uniform rate of net product of each technique.

To make clear the consequences of the choice of techniques on the basis of the two different orders, we started from the case in which the two orders coincide. In this case, the efficiency of the economic system declines when the activity level increases. But when the price-distribution order of efficiency gives an ordering of processes which is the reverse of that given by the physical order of efficiency, the economic system displays increasing efficiency with an increase of the activity level. Many other different cases, with constant or with alternating behaviours of efficiency, can be constructed on the basis of different relations between the prices–distribution and the physical order of efficiency.

To conclude: the main result of this analysis, which has been limited to uniperiodic and comparative uniperiodic situations, is to point out the relations existing among technologies, efficiency, and activity level by using the representation of the economic system based upon the scheme of global technologies.

The above approach could be extended to other issues connected to economic dynamics, technical progress and dynamic choice of technologies. But before going into very complex mathematical analysis with the global technology approach it must be taken into account that I have, also with a co-author, already dealt with these problems utilising the 'composite technology' approach with results which are very satisfactory. The global technology approach could therefore more properly be applied to other problems which suggest first of all uniperiodical and comparative uniperiodical consideration and then dynamic analysis.

Notes

* For the original Italian version of this essay see A. Quadrio Curzio, C. F. Manara and M. Faliva, 1987. Changes have been made here to the introduction, conclusions and references. Behind this study there are also some numerical simulations carried out in a research project supported, within the scheme of the National Grants 40%, 1987 by Ministero della Pubblica Istruzione at the Centre of Research in Economic Analysis at the Catholic University of Milan.

1 In the numbering of formulas, the main text and the appendices are considered separately.